Final Report: NAG 9-767

Concatenated Coding Using Trellis-Coded Modulation

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1 Research Description

1.1 Trellis Coded Modulation

In the late seventies and early eighties a technique known as Trellis Coded Modulation (TCM) was developed for providing spectrally efficient error correction coding (see [3], [4]). Instead of adding redundant information in the form of parity bits, redundancy is added at the modulation stage thereby increasing bandwidth efficiency. A digital communications system can be designed to use bandwidth-efficient multilevel/phase modulation schemes such as Amplitude Shift Keying (ASK), Phase Shift Keying (PSK), Differential Phase Shift Keying (DPSK) or Quadrature Amplitude Modulation (QAM). Performance gain can be achieved by increasing the number of signals over the corresponding uncoded system to compensate for the redundancy introduced by the code. A considerable amount of research and development has been devoted toward developing good TCM codes for severely bandlimited applications. More recently, the use of TCM for satellite and deep space communications applications has received increased attention.

A very important point to make is that TCM is the result of the combined "optimization" of both modulation and coding in an effort to increase performance. If modulation and coding are treated as separate and independent operations, then the use of very powerful (and hence large constraint length convolutional codes or large block length block codes) is required. By including the modulation scheme as an integral part of the encoding process it is possible to overcome the loss associated with expanding the signal set and to achieve significant coding gains without bandwidth expansion yet with relatively simple codes. The key to the combined modulation and coding approach is to map the signal points so the minimum Euclidean distance between transmitted sequences is maximized.

Figure 1 illustrates a PSK signal set that is expanded from 4 phases (uncoded) to 8 phases (coded) and also shows the Euclidean distances between phases. An example 4-state encoder that is used to map the information bit sequence into a corresponding set of phases is shown in Figure 2. Figure 2 also illustrates the trellis that is generated by the encoder and an example path through the trellis for a given sequence of information bits. Recall that for QPSK the information bits are grouped into pairs. The use of PSK based TCM codes is proposed as a starting point for this research since PSK provides constant-envelope modulation, which is often desirable for space and satellite communications systems.

The decoding of TCM codes can be accomplished with soft-decision Viterbi decoding. The decoding process is accomplished in two steps. Note that each branch in a trellis corresponds

Signal Constellation Expansion

QPSK Signal
Constellation

8-Phase PSK Signal
Constellation

$d_1 = 0.7654$

$d_2 = 1.4142$

$d_3 = 1.8478$

$d_4 = 2.0$

$s(t)=\cos(\omega_\tau + \theta)$
$i = 0,1,2,3$
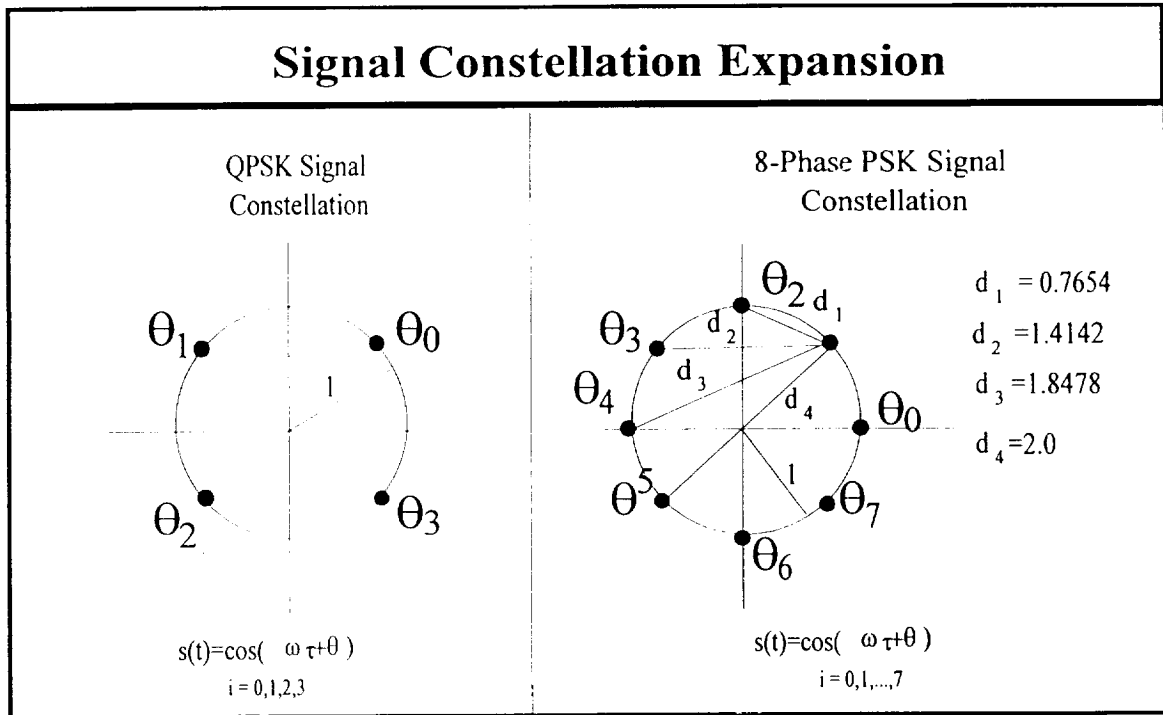
$s(t)=\cos(\omega_\tau + \theta)$
$i = 0,1,...,7$

Figure 1: The figure illustrates the expanded signal set for Quadrature Phase Shift Keyed (QPSK) modulation. The signal set is expanded from the transmission of 4 distinct phases to the transmission of 8 distinct phases. The figure also illustrates the (normalized) Euclidean distance between phases in the expanded signal set.

2

Four-State TCM Encoder

y

x | T | | T | (+) c2
                          c1 | 8-PSK | Phase
                               | Encoder | Output
                          c0

Trellis Diagram

Phase
Assignments

0426

1537

2604

3715

Information Sequence: 1,3,0,2

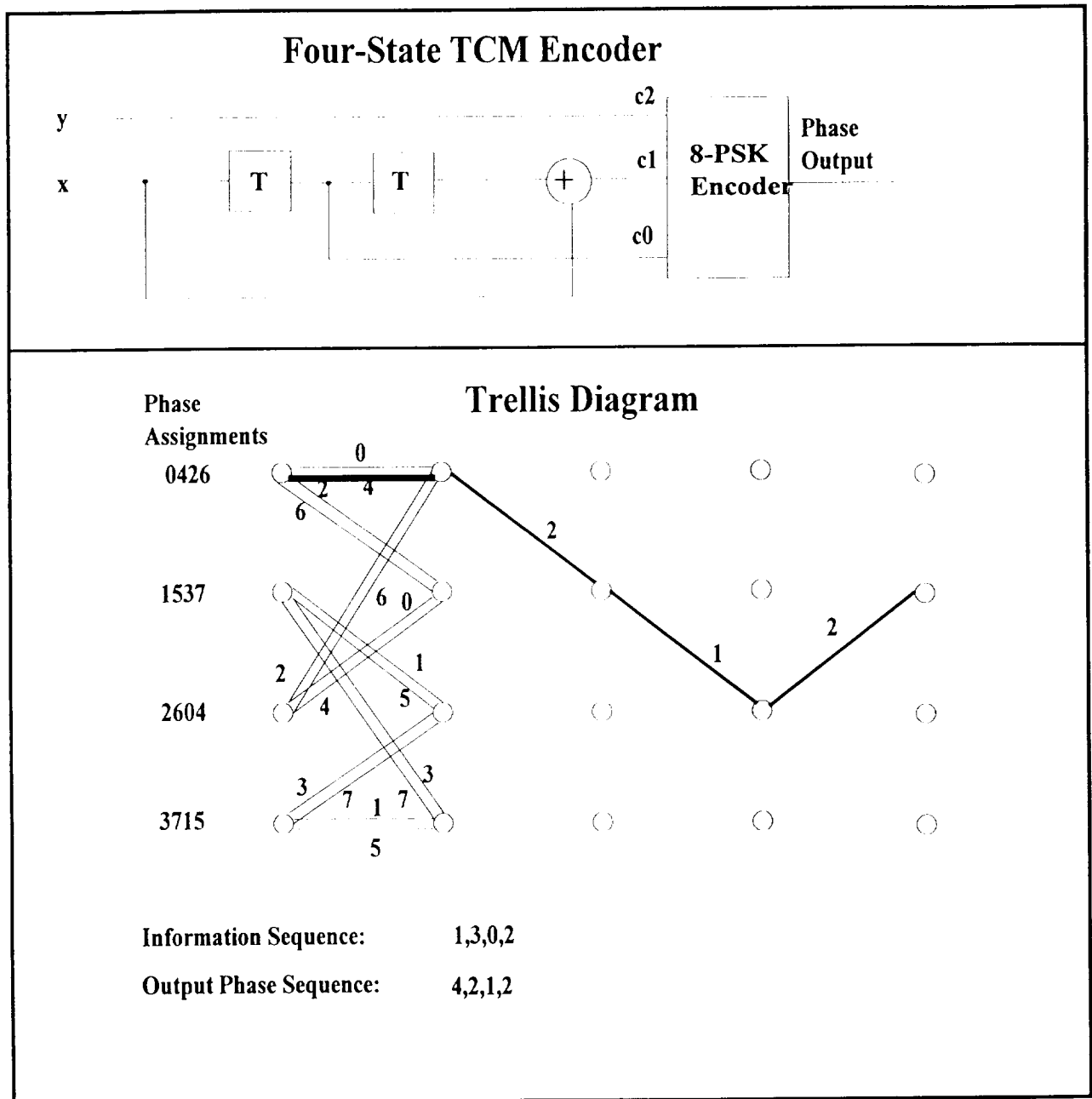Output Phase Sequence: 4,2,1,2

Figure 2: Illustration of a 4-state TCM encoder and trellis diagram. The darkened path indicates the transmitted phase sequence for the information sequence shown.

to a "subset" of signals. For example, in the trellis shown in Figure 2 there exist parallel state transition paths. Step one of the decoding procedure is to determine, by means of a Euclidean distance comparison, the best signal within the subset This process is called subset decoding. After subset decoding the signal point selected from each branch is used for another Euclidean distance comparison in order to select the minimum distance signal path through the trellis according to the Viterbi algorithm. It is important to note that the optimal path is based on the Euclidean distance between trellis phase assignments and the sequence of received (noisy) signals. For more details concerning the use of Viterbi decoding for TCM codes see [5].

1.2 Reed-Solomon Coding

The Reed-Solomon (RS) codes are an important class of linear block codes. The encoder for an RS code varies from the traditional binary encoder in that it operates on multiple bits rather than on a "bit-at-a-time" basis. The encoder for a size (n, k) RS code takes a group of m bits as a symbol element. The incoming binary stream is grouped into blocks that are km bits long and thus k symbols are grouped, with each symbol consisting of m bits. The encoding algorithm expands a block of k symbols to n symbols by adding $n - k$ redundant symbols. A popular value for m is 8 and in general the m-bit symbols are known as the byte size of the RS code.

The class of 8-bit RS codes are very powerful and in general they are maximum-distance separable codes. This means that RS codes are highly efficient in the use of redundancy for providing error correction. RS codes are designed to correct t-errors, where t is related to the number of parity symbols ($n - k = 2t$). It follows that the degree of bandwidth expansion associated with the use of RS codes is roughly given by the ratio $n/(n - k)$ and thus the choice of RS code involves a trade between error correction capability and bandwidth. RS codes are often used as the outer code in a concatenated scheme and it is worth noting that in many concatenated schemes it is the inner code that introduces the most redundancy. In our work, a TCM code is used as the inner code thereby achieving the greatest gain in spectral efficiency.

RS codes also offer flexibility in the choice of code-rates and message sizes allowing system optimization for a given application. Efficient decoding techniques are also well developed as described in [2]. Lin also discusses the good burst error correction capabilities associated with RS codes [2].

1.3 Concatenated Coding

Power savings and spectral efficiency are two conflicting requirements for communications

efficiency is improved dramatically since only the outer code requires additional parity bits. In fact, the outer code in a concatenated scheme sometimes requires significantly fewer parity bits than does the inner code and thus only a modest degree of bandwidth expansion is expected from this type of scheme.

Figure 3 presents a block diagram illustration of concatenated coding and interleaving.

Concatenated Coding

Information
Bit Stream

RS Encoder → Interleaver → TCM Encoder

Channel

Output
Bit Stream

RS Decoder ← Deinterleaver ← TCM Decoder

Bit Interleaving

Incoming Bits

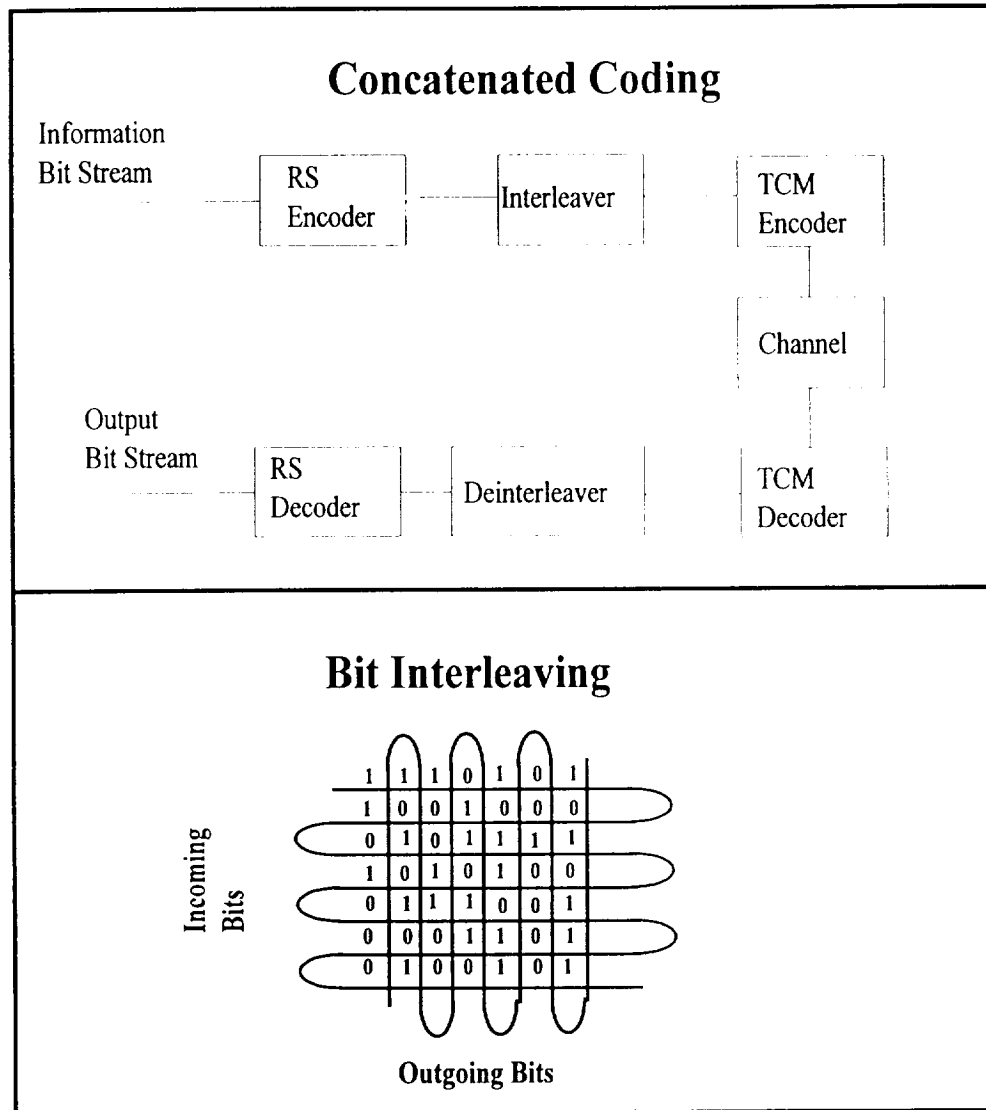| 1	1	1	0	1	0	1
1	0	0	1	0	0	0
0	1	0	1	1	1	1
1	0	1	0	1	0	0
0	1	1	1	0	0	1
0	0	0	1	1	0	1
0	1	0	0	1	0	1

Outgoing Bits

Figure 3: Block diagram illustration of Concatenated Coding with Interleaving

realistic simulation studies using realistic channel models and impairments. Figure 4 shows a block-diagram of a communication system developed under SPW. This illustrates the types of complexities that can be considered and we note that cross-talk, receiver and channel nonlinearites, fading and synchronization problems can be modelled and studied using the block-oriented approach featured by SPW. As a step toward meeting these future objectives the PI has attended a three-day workshop sponsored by the software developer/manufacturer.

3   Appendix A: References

References

[1] Simon Haykin, *Digital Communications*. John Wiley & Sons, New York. 1988

[2] Shu Lin and Daniel J. Costello, Jr.. *Error Control Coding. Fundamentals and Applications*. Prentice-Hall, Inc.. Englewood Cliffs, N.J. 1983.

[3] G. Ungerboeck,"Channel Coding with Multilevel/Phase Signals," *IEEE Transactions on Information Theory*, vol. IT-28, pp. 55-67, January 1982.

[4] G. Ungerboeck, "Trellis-Coded Modulation with Redundant Signal Sets, Parts I and II," *IEEE Communications Magazine*, vol. 25, pp. 5-21, Feb. 1987.

[5] John G. Proakis, *Digital Communications*, Second Edition, McGraw-Hill Book Company, 1989.

Concatenated Coding For Deep Space Communications
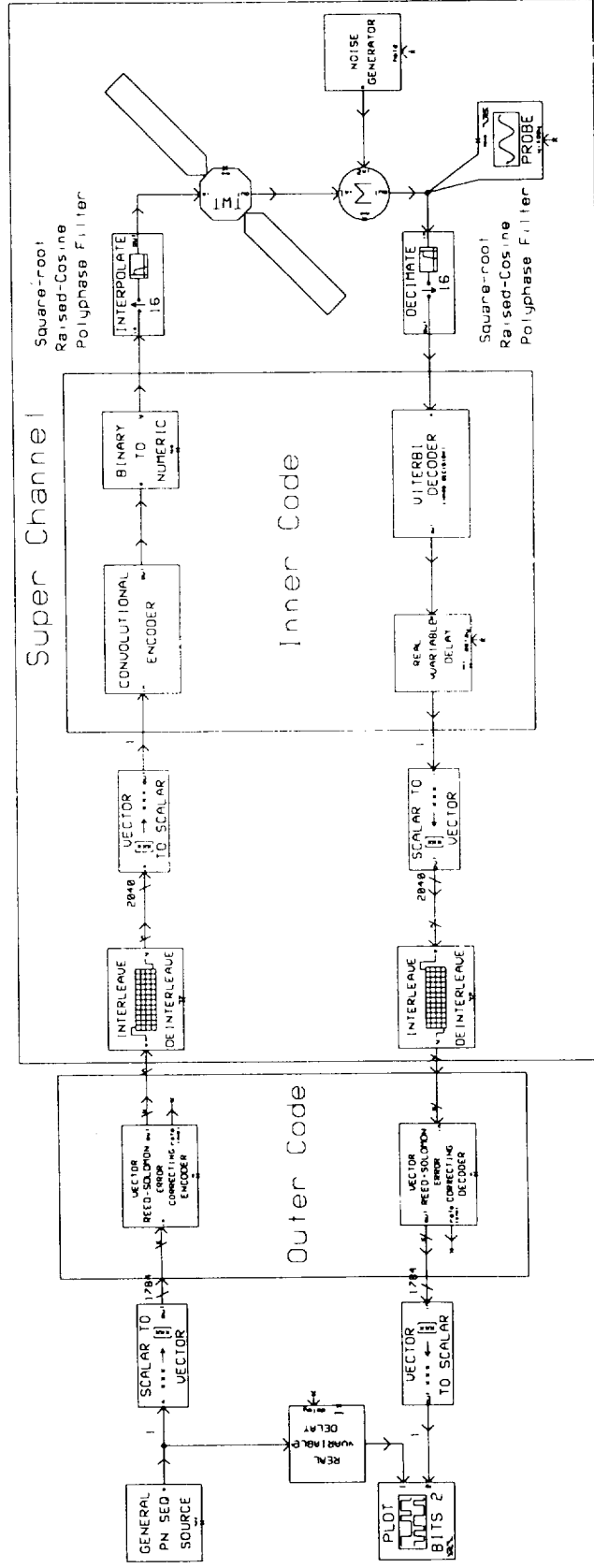
Reed_solomon Outer Code (223, 255)

Convolutional Inner Code (7, 1/2)

Figure 4: SPW Communication System Block Diagram